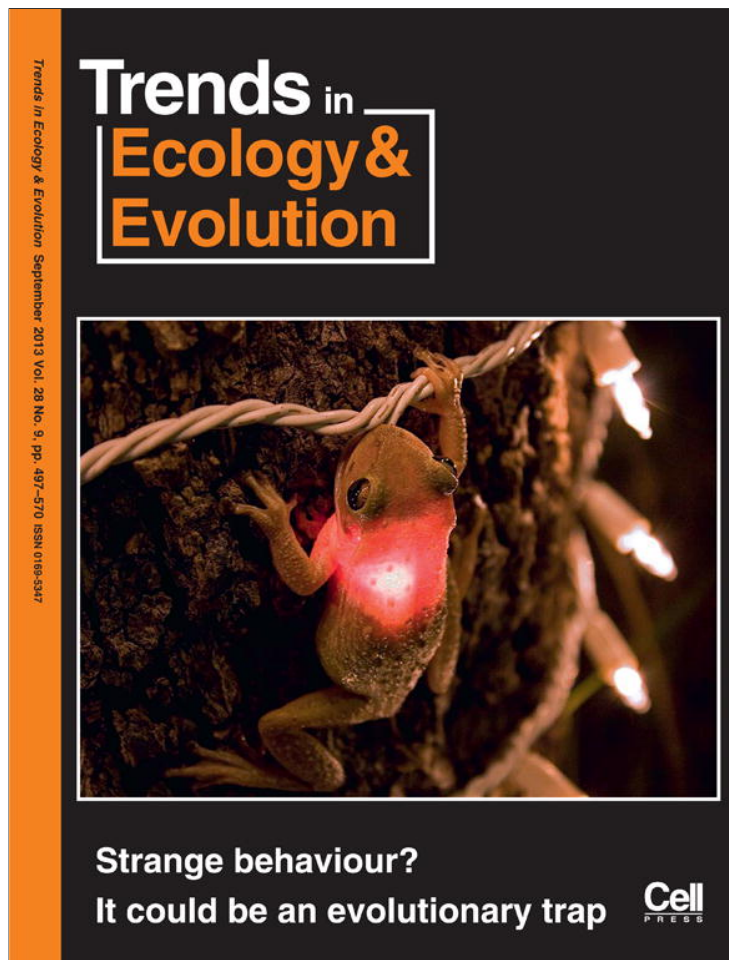


Provided for non-commercial research and education use.  
Not for reproduction, distribution or commercial use.



This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/authorsrights>

# Fractal geometry for animal biometrics: a response to Kühl and Burghardt

Roger Jovani<sup>1</sup>, Lorenzo Pérez-Rodríguez<sup>2,3</sup>, and François Mougeot<sup>4,5</sup>

<sup>1</sup> Department of Evolutionary Ecology, Estación Biológica de Doñana, CSIC, Avda. Americo Vesputio, s/n, 41092, Seville, Spain

<sup>2</sup> CIBIO, Centro de Investigação em Biodiversidade e Recursos Genéticos, Campus Agrário de Vairão, Rua Padre Armando Quintas, 4485-661 Vairão, Portugal

<sup>3</sup> Department of Evolutionary Ecology, Museo Nacional de Ciencias Naturales (MNCN-CSIC), José Gutiérrez Abascal 2, 28006, Madrid, Spain

<sup>4</sup> Instituto de Investigación en Recursos Cinegéticos, IREC (CSIC-UCLM-JCCM), Ronda de Toledo, s/n, 13005, Ciudad Real, Spain

<sup>5</sup> Estación Experimental de Zonas Áridas (EEZA-CSIC), Ctra. de Sacramento s/n La Cañada de San Urbano, 04120, Almería, Spain

In their recent article in *TREE*, Kühl and Burghardt [1] reviewed the emerging field of animal biometrics, but missed an opportunity to highlight an important and promising tool: fractal geometry. This tool has seldom been applied in the context of animal biometrics, but the few tentative examples reported so far have shown great potential to contribute to the development of this research field.

Fractals are mathematical sets that are self-similar across scales [2]. Natural examples of fractal-like behaviour include patterns that are built by recursive iteration, such as branching (e.g., roots, clouds, or lungs), where a small part ends up resembling the whole. Interestingly, fractal geometry can also describe patterns that are not strictly self-similar, but that show intricate, complex, and heterogeneous configurations. For this reason, fractal geometry offers new and valuable opportunities to describe and compare complex individual- or species-specific patterns.

As highlighted by Kühl and Burghardt [1], animal biometric tools characterise phenotypic appearance, provide ways to recognise phenotypes, and enable the profiling and description of individual behaviours. Some pioneering studies have shown the potential of fractal geometry for such tasks, by measuring the complexity of natural patterns through their fractal dimension (FD). For instance, FD can discriminate between butterfly species from their wing patterns [3], branchiopod morph types from the structure of their eggshells [4], mammalian species from their cranial suture patterns [5], and ammonoid taxa from their shell suture patterns [6]. Fractal analysis can also differentiate types of music and instruments [7], suggesting a potential use for the study of animal acoustic signals. Individual and collective animal behaviours have also been successfully characterised by fractal geometry. Examples include analyses of movement tracks [8], spatial distribution of individuals or nests [9,10], or burrow architecture of subterranean species [11]. Finally, black-and-white plumage patterns have also been described using fractal geometry, with variation in the FD pattern being shown experimentally to relate to individual quality [12].

As for most other tools used in animal biometrics, images of patterns require prior processing and standardising before fractal geometry analysis. Also, understanding what

variation in FD means for a particular pattern (i.e., which pattern properties cause variation in FD) may not be as straightforward as with tools designed to measure a particular feature of a pattern. However, the latter can turn into an advantage because understanding the meaning of FD for a particular pattern may unravel unknown features of the pattern under study. The main limitation may be that a similar FD for a given structure could be shared by different individuals or species. When that is the case, fractal dimension alone would be a bad choice, but could be used as one more trait for pattern matching or as an initial filter to discard improbable matches.

The advantages and opportunities that fractal geometry offers to animal biometric studies are several. First, measuring the FD of an object or pattern is relatively straightforward, and several methods (e.g., box-counting, dividers method [2,5,12]), and software tools are readily available. Second, fractal geometry captures the scaling properties of the object, and is scale invariant, something that is particularly useful when studying objects of different size, or comparing images taken at different distances. Third, the FD of an object or pattern provides a single value that summarises the way the pattern 'behaves' across scales. This could have biological relevance, because it mirrors the way individuals develop signalling traits in a coherent way across scales (e.g., from individual feathers to whole plumages for bird patterns), which could convey information (e.g., physiological state) about the individual displaying the pattern [12]. Fourth, it can be applied to a wide array of patterns, and software tools developed for the analysis of a given pattern in a given species could easily be adapted to the study of other patterns.

For all these reasons, we suggest that the field of animal biometrics would benefit greatly from incorporating fractal geometry analyses into the study of animal patterns, and we hope that this letter will stimulate further uses and applications within this research field.

## Acknowledgements

R.J. is supported by a 'Ramón y Cajal' (RYC-2009-03967) program from the Spanish Ministerio de Ciencia e Innovación (MICINN). L.P.-R. is supported by a postdoctoral contract from ICETA-CCDRN.

## References

- 1 Kühl, H.S. and Burghardt, T. (2013) Animal biometrics: quantifying and detecting phenotypic appearance. *Trends Ecol. Evol.* 28, 432–441

- 2 Mandelbrot, B.B. (1983) *The Fractal Geometry of Nature*. W.H. Freeman
- 3 Castrejón-Pita, A.A. *et al.* (2004) Fractal dimension in butterflies wings: a novel approach to understanding wing patterns? *J. Math. Biol.* 50, 584–594
- 4 Bruner, E. *et al.* (2013) Fractal analysis of the egg shell ornamentation in anostracans cysts: a quantitative approach to the morphological variations in *Chirocephalus ruffoi*. *Hydrobiologia* 705, 1–8
- 5 Gibert, J. and Palmqvist, P. (1995) Fractal analysis of the Orce skull sutures. *J. Hum. Evol.* 28, 561–575
- 6 Pérez-Claros, J.A. *et al.* (2002) First and second orders of suture complexity in ammonites: a new methodological approach using fractal analysis. *Math. Geol.* 34, 323–343
- 7 Hsu, K.J. (1993) Fractal geometry of music: from bird songs to Bach. In *Applications of Fractals and Chaos* (Crilly, A. *et al.*, eds), pp. 21–39, Springer-Verlag
- 8 Dicke, M. and Burrough, P.A. (1988) Using fractal dimension for characterising tortuosity of animal trails. *Physiol. Entomol.* 13, 393–398
- 9 Jovani, R. and Tella, J.L. (2007) Fractal bird nest distribution produces scale-free colony sizes. *Proc. R. Soc. B* 274, 2465–2469
- 10 Lander, M.E. *et al.* (2011) Spatial patterns and scaling behaviors of Steller sea lion (*Eumetopias jubatus*) distributions and their environment. *J. Theor. Biol.* 274, 74–83
- 11 Le Comber, S.C. *et al.* (2006) Burrow fractal dimension and foraging success in subterranean rodents: a simulation. *Behav. Ecol.* 17, 188–195
- 12 Pérez-Rodríguez, L. *et al.* (2013) Fractal geometry of a complex plumage trait reveals bird's quality. *Proc. R. Soc. B* 280, 20122783 <http://dx.doi.org/10.1098/rspb.2012.2783>

0169-5347/\$ – see front matter © 2013 Elsevier Ltd. All rights reserved.  
<http://dx.doi.org/10.1016/j.tree.2013.06.004> Trends in Ecology & Evolution, September 2013, Vol. 28, No. 9



# Fractal representation and recognition for animal biometrics: a reply to Jovani *et al.*

Hjalmar S. Kühl<sup>1</sup> and Tilo Burghardt<sup>2</sup>

<sup>1</sup> Department of Primatology, Max Planck Institute for Evolutionary Anthropology, Deutscher Platz 6, 04103 Leipzig, Germany

<sup>2</sup> Department of Computer Science, University of Bristol, Woodland Road, Bristol BS8 1UB, UK

In their letter to *TREE*, Jovani *et al.* [1] point toward fractal analysis as a promising tool for aiding animal biometrics. Despite the fact that few tentative real-world systems have adopted this approach, the letter adds very valuable further detail to our recent article [2]. In broad agreement with the authors, we would like to position the fractal approach within a slightly wider, also practical animal biometric context here and point to recent framework extensions suitable to tackle some of the currently open challenges.

Methodological contributions of fractal-based analysis relate to two key applications within animal biometrics, which are pattern representation and associative discrimination. The former refers to fractal descriptions of observed animal patterns by selected parameters, whereas the latter addresses the classification of these representations for the inference of animal traits and underlying processes. Both tasks may be underpinned by one fractal model. Figure 1 schematically illustrates this relationship.

Jovani *et al.* have listed various studies in which morphological traits and behaviors of different species have been characterized with regard to their Fractal Dimensions (FDs). Without doubt, fractal patterns are ubiquitous in nature, in animals and their traits, and have been shown to lend themselves as effective and efficient encodings of natural patterns and traits. However, the characterization of fractal animal patterns is only a first step. Few studies have attempted to make the critical link [3] between such fractal measures of phenotypic appearance and underlying biochemical, environmental, physiological, or genetic formation processes (Figure 1).

The systematic capture and representation of phenotypic appearance lies at the core of the newly emerging

field of phenomics [4]. This is where fractal-based animal biometrics can most likely make contributions toward a standardized representation of geometry in animal traits. Specific advantages offered by fractal analysis are a uniform and general framework for investigating biometric patterns over any number of spatial, temporal or other dimensions. In addition, fractal models support synthesis, yield compacted descriptions of self-similar pattern themes, and, as also mentioned by Jovani *et al.*, can be applied consistently over several pattern scales. Such self-similarity – that is, invariance at several distinct scales – does not imply scale invariance, which would guarantee invariance over all scales. This has practical implications and demands appropriate reprocessing and/or capture. For instance, when applied to imagery taken at different zoom levels, truly scale-invariant fractal descriptions are reliant on image samplings that resolve the discrete scales at which the self-similar modes reside. Otherwise, potentially characteristic fractal statistics such as FD may well deviate between different resolutions or scales in images.

In fact, none of the various FD measures can guarantee the provision of a stable characteristic function for biometric classification due to a lack of inherent bijection. That said, FD is nevertheless a valuable statistic on pattern data that can, as Jovani *et al.* hint, be used as an attribute to feed into the Machine Learning of classifiers trained to perform automatic inference of animal traits.

Finally, it is important to note that FD statistics are just one specific component within the framework of multifractal analysis [5] that expands the methodology to extract entire fractal spectra as pattern descriptors. Multifractal analysis can therefore increase the specificity of fractal statistics and help to get closer to producing characteristic functions that provide the unique link between phenotypic

Corresponding author: Kühl, H.S. ([kuehl@eva.mpg.de](mailto:kuehl@eva.mpg.de)).